SPECTRA OF TURBULENT VELOCITY AND TEMPERATURE FLUCTUATIONS AND THEIR CORRELATION IN THE CASE OF AIR FLOW THROUGH A CIRCULAR PIPE

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A procedure is shown for measuring the spectra of velocity and temperature fluctuations as well as of turbulent thermal and momentum fluxes. Results are shown of tests performed at a Reynolds number Re = 32,500.

The spectra of longitudinal and radial components of turbulent velocity fluctuations as well as the spectrum of temperature fluctuations were measured in an air stream through a circular pipe at a relative distance $2r/d = 0.5$ from the pipe axis, along with the correlation moments $\overline{vt}$ and $\overline{uv}$ for determining the density of turbulent thermal fluxes ($c_p\rho\overline{vt}$) and momentum fluxes ($\rho\overline{uv}$). The test conditions as well as the method of separating the fluctuation components and the variations of correlations $\overline{uv}$, $\overline{vt}$ have been described in [1], while individual test results are given in [1-3].

The spectra were measured with the aid of a bandpass filter. This filter was an audio and infra-audio frequency analyzer developed at the Lvov Polytechnic Institute. It consisted of a selective amplifier with a tuning T-bridge circuit in its feedback loop. The relative bandwidth of the filter was 3%. The shape and the width of the selectivity characteristic had been checked against signals from an audio generator. The filtered signal was amplified by a model U-4 amplifier and the dispersion was determined with a correlation meter.

For the purpose of low-frequency measurements (below 30 Hz), the signals from the thermoanemometer probe were prefiltered in the first-stage amplifier with a 30 Hz cutoff frequency. The time for integrating the signals in the correlation meter was set to 1 min at a quasiresonant signal frequency above 30 Hz and to 5 min at frequencies below 30 Hz. With a one-minute time interval at low frequencies there was a large discrepancy betwen repeated measurements of the signal dispersions at test frequency.

The same cruciform two-filament probe was used for measuring the spectra of longitudinal and radial velocity fluctuations, temperature fluctuations, and $\overline{uv}$, $\overline{vt}$ correlations. This, presumably, was to reduce the error in calculating the spectral correlation coefficients R^f_{uv}, R^f_{vt} and to eliminate the error due to an inaccurate determination of the time constants of different probes. The F_u and F_t spectra were determined from the sum spectrum of signals from the two probe filaments, while the F_v spectrum was determined from their difference spectrum. The F_{uv} spectrum of the momentum flux was calculated from the difference between dispersions of signals from the cruciform probe filaments at the extracted quasi-resonance frequency in an isothermal stream; the increment of this difference due to nonisothermality of the stream was treated as a contribution by the correlation moment $\overline{vt}$ at this quasiresonant frequency, i.e., as a quantity proportional to $F_{vt} \cdot \Delta f$. Such a way of determining the $\overline{vt}$ correlation spectrum was justified by the results of special measurements, indicating that at the low thermal flux density in our tests ($q = 0.142$ kW/m^2) the intensity of velocity fluctuations and the magnitude of the uv correlation moment had been respectively almost equal under isothermal and nonisothermal conditions of air flow. The filament superheat optimum for $\overline{vt}$ measurements was established on the basis of the considerations given in [1]. The thermal inertia of a filament was not fully compensated and the insufficient compensation was accounted for in the calculation of spectral densities from measured dispersions of narrow-band signals at frequencies 300 Hz and higher.

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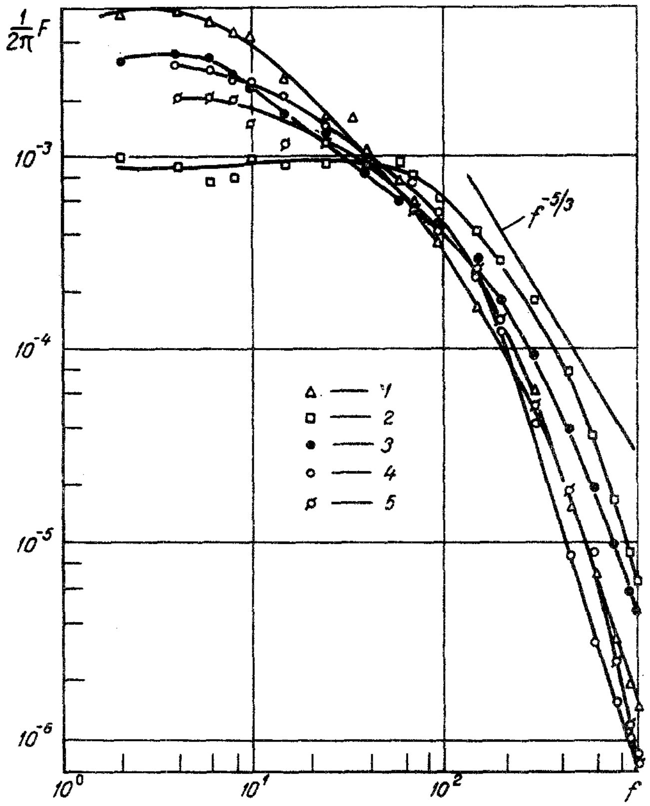

Fig. 1. Spectral densities of turbulent fluctuations F_u (1), F_v (2), F_t (3), of turbulent thermal fluxes $F_{\overline{uv}}$ (4), and of turbulent momentum fluxes (5).

The noise spectrum was also evaluated. It appeared insignificant within the test range of frequencies, but nevertheless it was corrected for in the determination of the F_u, F_v, and F_t spectra. In this way, the effect of noise became self-compensating in the spectral densities F_{uv} and F_{vt}. The resulting values of spectral densities were normalized into $\overline{u}^2$, $\overline{v}^2$, $\overline{t}^2$, $\overline{uv}$, and $\overline{vt}$ respectively.

Almost nowhere along the curves of spectral densities (Fig. 1) does a fluctuation spectrum vary according to the Kolmogorov (−5/3 power) law, which describes the inertial range of a spectrum. At f > 300 Hz the spectral density decreases faster than according to the −5/3 power law. This is, evidently, due to the low Reynolds number (Re = 32, 500) and, consequently, the fact that the rate of inertial cascading transfer of turbulence energy across the vortex spectrum is comparable with the dissipation of this energy in turbulent perturbations on the same scale.

The energy density spectrum of temperature fluctuations runs at an intermediate level between those of F_u and F_v, but the frequency spectrum of temperature fluctuations comes closest to the frequency spectrum of longitudinal velocity fluctuations (Fig. 1).

The correlation $\overline{uv}$ and $\overline{vt}$ spectra have a narrower frequency band with respect to energy than their component spectra of longitudinal and transverse velocity fluctuations (Fig. 1), which indicates a predominant effect of the large-scale component of turbulence in the transfer processes.

In order to compare the contributions of turbulent fluctuations at various frequencies to the momentum transfer and to the heat transfer, we have calculated the spectral coefficients of correlation between components u and v on the other:

$$R_{uv}^{f} = \frac{\overline{uv}^{f}}{(\overline{u^{2}}^{f}\,\overline{v^{2}}^{f})^{0.5}}, \quad R_{vt}^{f} = \frac{\overline{vt}^{f}}{(\overline{v^{2}}^{f}\,\overline{t^{2}}^{f})^{0.5}}.$$

The correlation coefficients of turbulent fluctuations vary widely within the frequency range under study here. At low frequencies (5-20 Hz) the correlation between velocity and temperature fluctuations is very high, coefficient R_{vt} (its absolute value) is near its maximum possible value equal to unity and is

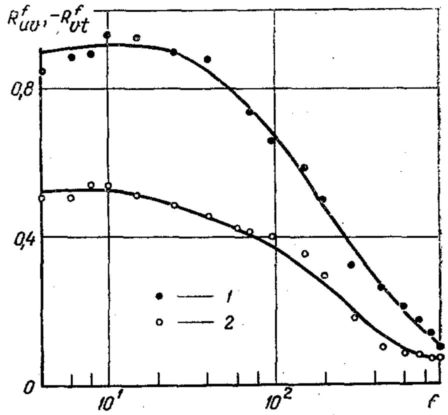

Fig. 2. Spectral coefficients of correlation between velocity and temperature fluctuations: R^f_{vt} (1), R^f_{uv} (2).

more than 1.5 times higher than R_{uv}. At higher frequencies the correlation between fluctuations becomes lower, with the values of both coefficients R_{uv} and R_{vt} converging (Fig. 2).

Inasmuch as large-scale turbulent vortices contribute most in low-frequency spectral measurements and small-scale turbulent vortices contribute most in high-frequency spectral measurements, this relation between the spectral correlation coefficients indicates that large-scale anisotropic turbulence, which produces bulk convection, contributes more to the heat transfer than to the momentum transfer.

The decrease of R^f_{uv} and R^f_{vt} with decreasing frequency indicates that the small-scale turbulence component tends to become isotropic in a turbulent shear flow even when the Reynolds number is low.

Measurements of R_{uv} in a flat channel [4], in a boundary layer at a flat plate [5], and in a jet [6] have also shown that in all such anisotropic modes of shear flow the correlation between velocity fluctuations decreases fast with rising frequency.

The spectrum of turbulent transverse heat flow was determined in [7] only for the case of atmospheric turbulence. The results of this study (measurements under various conditions of stratification and at various altitudes above ground level) have shown that the correlation between velocity and temperature fluctuations is high ($\approx$ 0.7) within the range of low wave numbers and decreases at higher wave numbers, i. e., R^f_{vt} varies with rising frequency in the same manner as in a channel flow.

NOTATION

d	is the pipe diameter;
r	is the distance from the pipe axis;
u	is the longitudinal component of turbulent velocity fluctuation;
v	is the radial component of turbulent velocity fluctuation;
t	is the temperature fluctuation;
$\overline{u^2}$, $\overline{v^2}$	are the dispersions of turbulent longitudinal and radial fluctuation;
$\overline{t^2}$	is the dispersion of turbulent temperature fluctuation;
$\overline{uv}$	is the correlation moment between longitudinal and radial velocity fluctuation;
$\overline{vt}$	is the correlation between radial velocity and temperature fluctuation;
$\overline{u^{2f}}$, $\overline{v^{2f}}$, $\overline{t^{2f}}$, $\overline{uv^f}$, $\overline{vt^f}$	are the dispersions of turbulent fluctuations and of correlation moments at a quasi-resonant frequency f;
Δf	is the pass band of the filter;
F_u, F_v, F_t, F_{uv}, F_{vt}	are the respective spectral densities of turbulent fluctuations and of correlation moments;
R^f_{uv}	is the spectral coefficient of correlation between longitudinal and radial fluctuation;
R^f_{vt}	is the spectral coefficient of correlation between radial velocity and temperature fluctuation;
q_w	is the thermal flux density at the pipe wall;
c_p	is the specific heat of air;
ρ	is the density of air;
Re	is the Reynolds number.

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